

A secret symmetry of the AdS/CFT S-matrix

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ABSTRACT: We find a new quantum Yangian symmetry of the AdS/CFT S-matrix, which complements the original $\mathfrak{su}(2|2)$ symmetry to $\mathfrak{gl}(2|2)$ and does not have a Lie algebra analog. Our finding is motivated by the Yangian double structure discovered at the classical level.

KEYWORDS: AdS-CFT Correspondence, Quantum Groups, Supersymmetric gauge theory.

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1. Introduction

One of the most important results in the study of the integrable spin chain inspired by AdS/CFT [1]¹ is that the relevant S-matrix [3] can be determined uniquely by the $\mathfrak{su}(2|2)$ Lie algebra symmetry of the problem, up to an overall dressing factor. This fact reduces the problem of the dynamics into a single function. The dressing factor satisfies the crossing symmetry constraint [4] originating from the underlying Hopf algebra structure [4–6]. A remarkable solution to Janik’s equation, reproducing the asymptotic behavior in the weak and strong coupling region [7, 8], was recently proposed by [9] and passed highly non-trivial checks [10].

In order to gain a deeper understanding of the hidden algebra responsible for such a structure,² it is desirable to understand all the symmetries of the model [13–15]. In [16] it was shown that the whole Lie algebra symmetry $\mathfrak{su}(2|2)$ is lifted to the infinite-dimensional Yangian symmetry by generalizing the standard formula of Drinfeld’s first realization of Yangians [17]:³

$$\Delta \hat{\mathcal{J}}^A = \hat{\mathcal{J}}^A \otimes 1 + 1 \otimes \hat{\mathcal{J}}^A + \frac{i}{2} \hbar f_{BC}^A \mathcal{J}^B \otimes \mathcal{J}^C, \quad S(\hat{\mathcal{J}}^A) = -\hat{\mathcal{J}}^A + \frac{i}{4} \hbar f_{BC}^A f_D^{BC} \mathcal{J}^D. \quad (1.1)$$

Since the Cartan matrix which is used to raise and lower the indices is degenerate, one appealed to $\mathfrak{sl}(2)$ automorphisms [18] which couple to the central charges.

In the study of the classical (near BMN) limit [19] it was noticed that one of the poles reveals the Casimir operator of the Lie algebra $\mathfrak{gl}(2|2)$, instead of the expected symmetry algebra $\mathfrak{su}(2|2)$. Although this fact suggests that the model has a larger symmetry of $\mathfrak{gl}(2|2)$, clearly the additional generator \mathfrak{I} which extends $\mathfrak{su}(2|2)$ to $\mathfrak{gl}(2|2)$

$$\mathfrak{I}|\phi^a\rangle = I|\phi^a\rangle, \quad \mathfrak{I}|\psi^\alpha\rangle = -I|\psi^\alpha\rangle, \quad (1.2)$$

is not a symmetry of the S-matrix, when equipped with a trivial coproduct.

¹We refer the reader to the reviews [2] and references therein.

²See [11] for a recent test of integrability in the near-flat space limit [12].

³Note that we can freely rescale \hbar at the present stage. We shall fix it later to a convenient value.

A possible resolution to this problem was subsequently found in [20] by rewriting the classical r-matrix in the form of a Yangian double (Drinfeld's second realization [21]). There, one found it necessary to include an infinite family of generators \mathfrak{J}_n to be able to factorize the classical r-matrix. Although the coefficient of the additional generators \mathfrak{J}_n remains non-trivial for the infinite-dimensional Yangian algebra, it actually vanishes for the classical Lie algebra at level $n = 0$.

Here we would like to continue the study of this additional symmetry. Since it does not have a Lie algebraic analog, we will call it a secret symmetry. Starting from the first higher Yangian level, the coproducts are usually non-trivial, and there is a chance for the new generators to be symmetries. In fact, we will find a quantum Yangian generator proportional to \mathfrak{J} , equipped with a coproduct that makes it a symmetry of the S-matrix. We shall exploit a hybrid version of the arguments adopted in [16] and [20]. Namely, we apply the formulas (1.1) together with the fact that the \mathfrak{J} operator couples to the central charge \mathfrak{C} [20]. We find

$$\Delta\widehat{\mathfrak{J}} = \widehat{\mathfrak{J}} \otimes 1 + 1 \otimes \widehat{\mathfrak{J}} + \frac{i}{2g}(\mathfrak{Q}^\alpha \mathfrak{U}^{-1} \otimes \mathfrak{S}^a_\alpha + \mathfrak{S}^a_\alpha \mathfrak{U}^{+1} \otimes \mathfrak{Q}^\alpha_a), \quad \mathfrak{S}(\widehat{\mathfrak{J}}) = -\widehat{\mathfrak{J}} + \frac{2i}{g}\mathfrak{C}, \quad (1.3)$$

where $\widehat{\mathfrak{J}}$ acts as \mathfrak{J} in (1.2), with the eigenvalue I replaced by \widehat{I} . In the main text we will show that the coproduct $\Delta\widehat{\mathfrak{J}}$ is a symmetry of the S-matrix if we choose

$$\widehat{I} = \frac{1}{4}(x^+ + x^- - 1/x^+ - 1/x^-). \quad (1.4)$$

The Hopf algebra structure associated to the Lie algebra generators \mathfrak{Q}^α_a , \mathfrak{S}^a_α and \mathfrak{C} in (1.3) is as described in [16]. The braiding factor \mathfrak{U} is a central element with eigenvalue $U = \sqrt{x^+/x^-}$. As from (1.1), it is easy to check that the defining relation of Hopf algebras $\mu \circ (\mathfrak{S} \otimes 1) \circ \Delta = \eta \circ \epsilon$ (μ is the algebra multiplication) holds for $\widehat{\mathfrak{J}}$, with the counit $\epsilon(\widehat{\mathfrak{J}}) = 0$.

A natural question arises whether the generator $\widehat{\mathfrak{J}}$ we found is a generator in Drinfeld's second realization, so that we can use it directly to construct the universal R-matrix. Taking the classical limit of this operator, which essentially consists in replacing x^\pm by the classical spectral parameter x [8, 19], we find

$$\widehat{I} \rightarrow \frac{1}{2}(x - 1/x). \quad (1.5)$$

This coincides with the expression of the first level generator $\mathfrak{J}_{n=1}$ found in [20], where Drinfeld's second realization was employed. Therefore, it appears natural to expect that the operator $\widehat{\mathfrak{J}}$ is in the second realization. However, as we will argue below, this is not likely to be the case.

Since the generators in Drinfeld's second realization are the ones directly appearing in the universal form of the R-matrix, they have to be consistent with the crossing equation found by Janik [4]:

$$(\mathcal{C}^{-1} \otimes 1)[\mathcal{R}(1/x_1^\pm, x_2^\pm)]^{st_1}(\mathcal{C} \otimes 1)\mathcal{R}(x_1^\pm, x_2^\pm) = 1, \quad (1.6)$$

where \mathcal{C} is a (bosonic) charge conjugation matrix and st_1 denotes supertransposition in the first entry. This equation originates from combining the knowledge of the crossing

symmetry transformation of the Lie algebra ($n = 0$) generators with the fundamental property $(S \otimes 1)\mathcal{R} = \mathcal{R}^{-1}$ of quasi-triangular Hopf algebras. On the other hand, if we assume that the universal R-matrix \mathcal{R} admits some expansion in powers of the Yangian generators \mathcal{J}_n , (1.6) leads to the expectation that all \mathcal{J}_n 's have to satisfy the antipode relation

$$S(\mathcal{J}_n(x^\pm)) = \mathcal{C}^{-1}[\mathcal{J}_n(1/x^\pm)]^{st}\mathcal{C}, \quad (1.7)$$

with one and the same charge conjugation \mathcal{C} . Apparently, although the operator $\widehat{\mathcal{J}}$ satisfies the relation $\mathcal{C}^{-1}[\widehat{\mathcal{J}}(1/x^\pm)]^{st}\mathcal{C} = -\widehat{\mathcal{J}}(x^\pm)$, it does not satisfy (1.7). Hence, it is difficult to expect this operator to be in the second realization.⁴

As a side remark, we notice that all the generators found in [20] satisfy the classical antipode relation

$$S(\mathcal{J}_n(x)) = \mathcal{C}_0^{-1}[\mathcal{J}_n(1/x)]^{st}\mathcal{C}_0, \quad (1.8)$$

where \mathcal{C}_0 is the classical charge conjugation, and the classical antipode is given by

$$S(\mathcal{J}_n(x)) = -\mathcal{J}_n(x). \quad (1.9)$$

The explicit expression of \mathcal{C}_0 is not necessary to show that the classical antipode relation (1.8) holds for the infinite tower of generators, provided the relation is satisfied by the Lie algebra generators at level $n = 0$ [4, 6, 14].

In the last part of the introduction, let us recall for convenience the action of the supercharges⁵

$$\begin{aligned} \Omega_a^\alpha|\phi^b\rangle &= a\delta_a^b|\psi^\alpha\rangle, & \Omega_a^\alpha|\psi^\beta\rangle &= b\epsilon^{\alpha\beta}\epsilon_{ab}|\phi^b\rangle, \\ \mathfrak{S}_\alpha^a|\phi^b\rangle &= c\epsilon^{ab}\epsilon_{\alpha\beta}|\psi^\beta\rangle, & \mathfrak{S}_\alpha^a|\psi^\beta\rangle &= d\delta_\alpha^\beta|\phi^a\rangle, \end{aligned} \quad (1.10)$$

with a, b, c and d parameterized by

$$a = \sqrt{g}\gamma, \quad b = \sqrt{g}\frac{\alpha}{\gamma}\left(1 - \frac{x^+}{x^-}\right), \quad c = \sqrt{g}\frac{i\gamma}{\alpha x^+}, \quad d = \sqrt{g}\frac{x^+}{i\gamma}\left(1 - \frac{x^-}{x^+}\right), \quad (1.11)$$

and the R-matrix [3, 18] that we will use in this paper:

$$\begin{aligned} \mathcal{R}_{12}|\phi_1^a\phi_2^b\rangle &= \frac{1}{2}(A_{12} - B_{12})|\phi_1^a\phi_2^b\rangle + \frac{1}{2}(A_{12} + B_{12})|\phi_1^b\phi_2^a\rangle + \frac{1}{2}C_{12}\epsilon^{ab}\epsilon_{\alpha\beta}|\psi_1^\alpha\psi_2^\beta\rangle, \\ \mathcal{R}_{12}|\psi_1^\alpha\psi_2^\beta\rangle &= -\frac{1}{2}(D_{12} - E_{12})|\psi_1^\alpha\psi_2^\beta\rangle - \frac{1}{2}(D_{12} + E_{12})|\psi_1^\beta\psi_2^\alpha\rangle - \frac{1}{2}F_{12}\epsilon^{\alpha\beta}\epsilon_{ab}|\phi_1^a\phi_2^b\rangle, \\ \mathcal{R}_{12}|\phi_1^a\psi_2^\beta\rangle &= G_{12}|\phi_1^a\psi_2^\beta\rangle + H_{12}|\psi_1^\beta\phi_2^a\rangle, \\ \mathcal{R}_{12}|\psi_1^\alpha\phi_2^b\rangle &= L_{12}|\psi_1^\alpha\phi_2^b\rangle + K_{12}|\phi_1^b\psi_2^\alpha\rangle. \end{aligned} \quad (1.12)$$

⁴Nevertheless, we still find it a little bizarre that (1.7) is not satisfied by this new symmetry. This issue certainly deserves further investigation, on which we reserve to come back in the future.

⁵We follow the notation of [18]. In particular, note that g in [18] is related to the one in [3] by $g_{[18]} = g_{[3]}/\sqrt{2}$.

The functions A_{12}, B_{12}, \dots are given by

$$\begin{aligned}
 A_{12} &= \frac{x_2^+ - x_1^-}{x_2^- - x_1^+}, \\
 B_{12} &= \frac{x_2^+ - x_1^-}{x_2^- - x_1^+} \left(1 - 2 \frac{1 - 1/x_1^+ x_2^-}{1 - 1/x_1^+ x_2^+} \frac{x_2^- - x_1^-}{x_2^+ - x_1^+} \right), \\
 C_{12} &= \frac{2\gamma_1 \gamma_2 U_2}{\alpha x_1^+ x_2^+} \frac{1}{1 - 1/x_1^+ x_2^+} \frac{x_2^- - x_1^-}{x_2^+ - x_1^+}, \\
 D_{12} &= -\frac{U_2}{U_1}, \\
 E_{12} &= -\frac{U_2}{U_1} \left(1 - 2 \frac{1 - 1/x_1^- x_2^+}{1 - 1/x_1^- x_2^-} \frac{x_2^+ - x_1^+}{x_2^- - x_1^-} \right), \\
 F_{12} &= -\frac{2\alpha(x_1^+ - x_1^-)(x_2^+ - x_2^-)}{\gamma_1 \gamma_2 U_1 x_1^- x_2^-} \frac{1}{1 - 1/x_1^- x_2^-} \frac{x_2^+ - x_1^+}{x_2^- - x_1^-}, \\
 G_{12} &= \frac{1}{U_1} \frac{x_2^+ - x_1^+}{x_2^- - x_1^-}, \\
 H_{12} &= \frac{\gamma_1 U_2}{\gamma_2 U_1} \frac{x_2^+ - x_2^-}{x_2^- - x_1^+}, \\
 L_{12} &= U_2 \frac{x_2^- - x_1^-}{x_2^- - x_1^+}, \\
 K_{12} &= \frac{\gamma_2}{\gamma_1} \frac{x_1^+ - x_1^-}{x_2^- - x_1^+}, \tag{1.13}
 \end{aligned}$$

where $U = \sqrt{x^+/x^-}$ and x^\pm satisfy the relation $x^+ + 1/x^+ - x^- - 1/x^- = i/g$.

2. Secret symmetry

In this section, we would like to show that the coproduct of the additional operator

$$\Delta \widehat{\mathcal{J}} = \widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}} + \frac{i}{2g} (\Omega^\alpha_a \mathcal{U}^{-1} \otimes \mathfrak{S}^a_\alpha + \mathfrak{S}^a_\alpha \mathcal{U}^{+1} \otimes \Omega^\alpha_a), \tag{2.1}$$

is an exact symmetry of the S-matrix:

$$[\Delta \widehat{\mathcal{J}}, \mathcal{S}] = 0. \tag{2.2}$$

This equation can be expressed in terms of the R-matrix $\mathcal{R} = \Pi \mathcal{S}$ (Π is the graded permutation operator) as

$$[\widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}}, \mathcal{R}] = \frac{i}{2g} [\{\Omega, \mathfrak{S}\}_\otimes \mathcal{R} + \mathcal{R} \{\Omega, \mathfrak{S}\}_\otimes]. \tag{2.3}$$

Here the expression on the right-hand-side is a bookkeeping notation:

$$\begin{aligned}
 \{\Omega, \mathfrak{S}\}_\otimes \mathcal{R} &= (\Omega^\alpha_a \otimes \mathcal{U}^{+1} \mathfrak{S}^a_\alpha + \mathfrak{S}^a_\alpha \otimes \mathcal{U}^{-1} \Omega^\alpha_a) \mathcal{R}, \\
 \mathcal{R} \{\Omega, \mathfrak{S}\}_\otimes &= \mathcal{R} (\Omega^\alpha_a \mathcal{U}^{-1} \otimes \mathfrak{S}^a_\alpha + \mathfrak{S}^a_\alpha \mathcal{U}^{+1} \otimes \Omega^\alpha_a). \tag{2.4}
 \end{aligned}$$

Note that the braiding factor of the first (second) line is in the second (first) entry.

The first term on the right-hand-side of (2.3) is given by

$$\begin{aligned}
 \{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} |\phi_1^a \phi_2^b\rangle &= -B_{12}(a_1 c_2 U_2 + c_1 a_2 U_2^{-1}) \epsilon^{ab} \epsilon_{\alpha\beta} |\psi_1^\alpha \psi_2^\beta\rangle \\
 &\quad + C_{12}(-b_1 d_2 U_2 - d_1 b_2 U_2^{-1}) (|\phi_1^a \phi_2^b\rangle - |\phi_1^b \phi_2^a\rangle), \\
 \{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} |\psi_1^\alpha \psi_2^\beta\rangle &= E_{12}(-b_1 d_2 U_2 - d_1 b_2 U_2^{-1}) \epsilon^{\alpha\beta} \epsilon_{ab} |\phi_1^a \phi_2^b\rangle \\
 &\quad - F_{12}(a_1 c_2 U_2 + c_1 a_2 U_2^{-1}) (|\psi_1^\alpha \psi_2^\beta\rangle - |\psi_1^\beta \psi_2^\alpha\rangle), \\
 \{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} |\phi_1^a \psi_2^\beta\rangle &= G_{12}(a_1 d_2 U_2 + c_1 b_2 U_2^{-1}) |\psi_1^\beta \phi_2^a\rangle + H_{12}(-b_1 c_2 U_2 - d_1 a_2 U_2^{-1}) |\phi_1^a \psi_2^\beta\rangle, \\
 \{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} |\psi_1^\alpha \phi_2^b\rangle &= L_{12}(-b_1 c_2 U_2 - d_1 a_2 U_2^{-1}) |\phi_1^b \psi_2^\alpha\rangle + K_{12}(a_1 d_2 U_2 + c_1 b_2 U_2^{-1}) |\psi_1^\alpha \phi_2^b\rangle,
 \end{aligned} \tag{2.5}$$

while the second term is

$$\begin{aligned}
 \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes} |\phi_1^a \phi_2^b\rangle &= (a_1 U_1^{-1} c_2 + c_1 U_1 a_2) (E_{12} \epsilon^{ab} \epsilon_{\alpha\beta} |\psi_1^\alpha \psi_2^\beta\rangle - F_{12} (|\phi_1^a \phi_2^b\rangle - |\phi_1^b \phi_2^a\rangle)), \\
 \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes} |\psi_1^\alpha \psi_2^\beta\rangle &= (-b_1 U_1^{-1} d_2 - d_1 U_1 b_2) (-B_{12} \epsilon^{\alpha\beta} \epsilon_{ab} |\phi_1^a \phi_2^b\rangle + C_{12} (|\psi_1^\alpha \psi_2^\beta\rangle - |\psi_1^\beta \psi_2^\alpha\rangle)), \\
 \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes} |\phi_1^a \psi_2^\beta\rangle &= (a_1 U_1^{-1} d_2 + c_1 U_1 b_2) (L_{12} |\psi_1^\beta \phi_2^a\rangle + K_{12} |\phi_1^a \psi_2^\beta\rangle), \\
 \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes} |\psi_1^\alpha \phi_2^b\rangle &= (-b_1 U_1^{-1} c_2 - d_1 U_1 a_2) (G_{12} |\phi_1^b \psi_2^\alpha\rangle + H_{12} |\psi_1^\alpha \phi_2^b\rangle).
 \end{aligned} \tag{2.6}$$

Using the relations

$$\begin{aligned}
 -B_{12}(a_1 c_2 U_2 + c_1 a_2 U_2^{-1}) + E_{12}(a_1 U_1^{-1} c_2 + c_1 U_1 a_2) &= 2igC_{12}(\widehat{I}_1 + \widehat{I}_2), \\
 C_{12}(-b_1 d_2 U_2 - d_1 b_2 U_2^{-1}) - F_{12}(a_1 U_1^{-1} c_2 + c_1 U_1 a_2) &= 0, \\
 E_{12}(-b_1 d_2 U_2 - d_1 b_2 U_2^{-1}) - B_{12}(-b_1 U_1^{-1} d_2 - d_1 U_1 b_2) &= 2igF_{12}(\widehat{I}_1 + \widehat{I}_2), \\
 -F_{12}(a_1 c_2 U_2 + c_1 a_2 U_2^{-1}) + C_{12}(-b_1 U_1^{-1} d_2 - d_1 U_1 b_2) &= 0, \\
 G_{12}(a_1 d_2 U_2 + c_1 b_2 U_2^{-1}) + L_{12}(a_1 U_1^{-1} d_2 + c_1 U_1 b_2) &= 4igH_{12}(\widehat{I}_1 - \widehat{I}_2), \\
 H_{12}(-b_1 c_2 U_2 - d_1 a_2 U_2^{-1}) + K_{12}(a_1 U_1^{-1} d_2 + c_1 U_1 b_2) &= 0, \\
 L_{12}(-b_1 c_2 U_2 - d_1 a_2 U_2^{-1}) + G_{12}(-b_1 U_1^{-1} c_2 - d_1 U_1 a_2) &= -4igK_{12}(\widehat{I}_1 - \widehat{I}_2), \\
 K_{12}(a_1 d_2 U_2 + c_1 b_2 U_2^{-1}) + H_{12}(-b_1 U_1^{-1} c_2 - d_1 U_1 a_2) &= 0,
 \end{aligned} \tag{2.7}$$

if we define $\widehat{I}_{1(2)}$ as

$$\widehat{I} = \frac{1}{4}(x^+ + x^- - 1/x^+ - 1/x^-), \tag{2.8}$$

we find

$$\begin{aligned}
 [\{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} + \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes}] |\phi_1^a \phi_2^b\rangle &= 2igC_{12}(\widehat{I}_1 + \widehat{I}_2) \epsilon^{ab} \epsilon_{\alpha\beta} |\psi_1^\alpha \psi_2^\beta\rangle, \\
 [\{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} + \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes}] |\psi_1^\alpha \psi_2^\beta\rangle &= 2igF_{12}(\widehat{I}_1 + \widehat{I}_2) \epsilon^{\alpha\beta} \epsilon_{ab} |\phi_1^a \phi_2^b\rangle, \\
 [\{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} + \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes}] |\phi_1^a \psi_2^\beta\rangle &= 4igH_{12}(\widehat{I}_1 - \widehat{I}_2) |\psi_1^\beta \phi_2^a\rangle, \\
 [\{\Omega, \mathfrak{S}\}_{\otimes} \mathcal{R} + \mathcal{R} \{\Omega, \mathfrak{S}\}_{\otimes}] |\psi_1^\alpha \phi_2^b\rangle &= -4igK_{12}(\widehat{I}_1 - \widehat{I}_2) |\phi_1^b \psi_2^\alpha\rangle.
 \end{aligned} \tag{2.9}$$

Most of the computations in (2.7) are tedious but straightforward, except the first and the third line with B_{12} and E_{12} , which require a more elaborated treatment. The

easiest way to compute them is to separately deal with the ‘1’ term and the ‘−2’ term in the parenthesis of B_{12} and E_{12} in (1.13). It is not difficult to find that the ‘−2’ term is proportional to C_{12} and F_{12} respectively. For the ‘1’ term, the following identity is useful

$$x_2^+ x_2^- - x_1^+ x_1^- = 2 \frac{x_2^- - x_1^-}{1 - 1/x_1^+ x_2^+} (\widehat{I}_1 + \widehat{I}_2), \quad (2.10)$$

which can be obtained by multiplying the identity

$$x_2^+ x_2^- - x_1^+ x_1^- = \frac{1}{2} ((x_2^+ + x_1^+) (x_2^- - x_1^-) + (x_2^- + x_1^-) (x_2^+ - x_1^+)), \quad (2.11)$$

by the quantity $(1 - 1/x_1^+ x_2^+) / (x_2^- - x_1^-) = (1 - 1/x_1^- x_2^-) / (x_2^+ - x_1^+)$.

Comparing (2.9) with

$$\begin{aligned} [\widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}}, \mathcal{R}] |\phi_1^a \phi_2^b\rangle &= 2(-\widehat{I}_1 - \widehat{I}_2) (C_{12}/2) \epsilon^{ab} \epsilon_{\alpha\beta} |\psi_1^\alpha \psi_2^\beta\rangle, \\ [\widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}}, \mathcal{R}] |\psi_1^\alpha \psi_2^\beta\rangle &= 2(\widehat{I}_1 + \widehat{I}_2) (-F_{12}/2) \epsilon^{\alpha\beta} \epsilon_{ab} |\phi_1^a \phi_2^b\rangle, \\ [\widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}}, \mathcal{R}] |\phi_1^a \psi_2^\beta\rangle &= 2(-\widehat{I}_1 + \widehat{I}_2) H_{12} |\psi_1^\beta \phi_2^a\rangle, \\ [\widehat{\mathcal{J}} \otimes 1 + 1 \otimes \widehat{\mathcal{J}}, \mathcal{R}] |\psi_1^\alpha \phi_2^b\rangle &= 2(\widehat{I}_1 - \widehat{I}_2) K_{12} |\phi_1^b \psi_2^\alpha\rangle, \end{aligned} \quad (2.12)$$

we eventually find that (2.3) holds.

3. Conclusion

We have found a new quantum Yangian symmetry of the AdS/CFT S-matrix, which complements the original $\mathfrak{su}(2|2)$ symmetry to $\mathfrak{gl}(2|2)$ and does not have a Lie algebra analog. Using this novel symmetry we can generate several new ones. For example, we can compute the commutators $[\Delta \widehat{\mathcal{J}}, \Delta \mathcal{Q}^\alpha_b]$ and $[\Delta \widehat{\mathcal{J}}, \Delta \mathcal{S}^a_\beta]$. Taking the linear combination with the known invariances of the S-matrix $\Delta \widehat{\mathcal{Q}}^\alpha_b$ and $\Delta \widehat{\mathcal{S}}^a_\beta$ discovered in [16], we find new symmetries:⁶ ($\hbar = 1/(2g)$)

$$\begin{aligned} \Delta \mathcal{Q}^\alpha_{b,1} &= \mathcal{Q}^\alpha_{b,1} \otimes 1 + \mathcal{U}^{+1} \otimes \mathcal{Q}^\alpha_{b,1} + i\hbar (\mathcal{L}^\alpha_\gamma \mathcal{U}^{+1} \otimes \mathcal{Q}^\gamma_b + \mathcal{K}^c_b \mathcal{U}^{+1} \otimes \mathcal{Q}^\alpha_c + \mathcal{C} \mathcal{U}^{+1} \otimes \mathcal{Q}^\alpha_b \\ &\quad - \mathcal{Q}^\gamma_b \otimes \mathcal{L}^\alpha_\gamma - \mathcal{Q}^\alpha_c \otimes \mathcal{K}^c_b - \mathcal{Q}^\alpha_b \otimes \mathcal{C}), \\ \Delta \mathcal{Q}^\alpha_{b,-1} &= \mathcal{Q}^\alpha_{b,-1} \otimes 1 + \mathcal{U}^{+1} \otimes \mathcal{Q}^\alpha_{b,-1} + i\hbar (-\epsilon^{\alpha\gamma} \epsilon_{bd} \mathcal{P} \mathcal{U}^{-1} \otimes \mathcal{S}^d_\gamma + \epsilon^{\alpha\gamma} \epsilon_{bd} \mathcal{S}^d_\gamma \mathcal{U}^{+2} \otimes \mathcal{P}), \\ \Delta \mathcal{S}^a_{\beta,1} &= \mathcal{S}^a_{\beta,1} \otimes 1 + \mathcal{U}^{-1} \otimes \mathcal{S}^a_{\beta,1} + i\hbar (-\mathcal{K}^a_c \mathcal{U}^{-1} \otimes \mathcal{S}^c_\beta - \mathcal{L}^\gamma_\beta \mathcal{U}^{-1} \otimes \mathcal{S}^a_\gamma - \mathcal{C} \mathcal{U}^{-1} \otimes \mathcal{S}^a_\beta \\ &\quad + \mathcal{S}^c_\beta \otimes \mathcal{K}^a_c + \mathcal{S}^a_\gamma \otimes \mathcal{L}^\gamma_\beta + \mathcal{S}^a_\beta \otimes \mathcal{C}), \\ \Delta \mathcal{S}^a_{\beta,-1} &= \mathcal{S}^a_{\beta,-1} \otimes 1 + \mathcal{U}^{-1} \otimes \mathcal{S}^a_{\beta,-1} + i\hbar (\epsilon^{ac} \epsilon_{\beta\delta} \mathcal{R} \mathcal{U}^{+1} \otimes \mathcal{Q}^\delta_c - \epsilon^{ac} \epsilon_{\beta\delta} \mathcal{Q}^\delta_c \mathcal{U}^{-2} \otimes \mathcal{R}), \end{aligned} \quad (3.1)$$

where $\mathcal{Q}^\alpha_{b,\pm 1}$ and $\mathcal{S}^a_{\beta,\pm 1}$ are defined as

$$\begin{aligned} \mathcal{Q}^\alpha_{b,1} &= \mathcal{Q}^\alpha_b (u \Pi_b + v \Pi_f), & \mathcal{Q}^\alpha_{b,-1} &= \mathcal{Q}^\alpha_b (v \Pi_b + u \Pi_f), \\ \mathcal{S}^a_{\beta,1} &= \mathcal{S}^a_\beta (v \Pi_b + u \Pi_f), & \mathcal{S}^a_{\beta,-1} &= \mathcal{S}^a_\beta (u \Pi_b + v \Pi_f), \end{aligned} \quad (3.2)$$

⁶As a check, we have explicitly verified that one of them, namely $\Delta \mathcal{Q}^\alpha_{b,1}$, is an exact symmetry of the S-matrix: $[\Delta \mathcal{Q}^\alpha_{b,1}, \mathcal{S}] = 0$.

with Π_b (Π_f) being the projector to bosons (fermions) and

$$u = \frac{1}{2}(x^+ + x^-), \quad v = \frac{1}{2}(1/x^+ + 1/x^-). \quad (3.3)$$

The antipodes are given by $S(\mathfrak{Q}^{\alpha}_{b,\pm 1}) = -\mathfrak{U}^{-1}\mathfrak{Q}^{\alpha}_{b,\pm 1}$ and $S(\mathfrak{S}^{\alpha}_{\beta,\pm 1}) = -\mathfrak{U}^{+1}\mathfrak{S}^{\alpha}_{\beta,\pm 1}$. The names we have chosen for these operators are inspired by their classical limit, and by the fact that they fulfill the crossing symmetry property (1.7) at the quantum level. Further commutators such as $\{\Delta\mathfrak{Q}^{\alpha}_{b,1}, \Delta\mathfrak{S}^{\alpha}_{\beta}\}$ or $\{\Delta\mathfrak{Q}^{\alpha}_b, \Delta\mathfrak{S}^{\alpha}_{\beta,1}\}$ actually generate the coproducts $\Delta\widehat{\mathfrak{R}}^a_b$ and $\Delta\widehat{\mathfrak{L}}^{\alpha}_{\beta}$ of [16], which satisfy the crossing symmetry relation at the quantum level, and coincide with the generators $\Delta\mathfrak{R}^a_{b,1}$ and $\Delta\mathfrak{L}^{\alpha}_{\beta,1}$ found in [20] in the classical limit.

Let us list several further directions related to the results we have presented here.

It is always crucial to study the symmetries when investigating the correspondence between two theories. The primary question which comes to mind is of course to see whether and how the new one we have found in this paper will appear in the string worldsheet theory.

An interesting application might be related to the string interaction vertices. In discussing the relation [22] between matrix string theory and light-cone string field theory on the flat space, the supersymmetry algebra on both sides plays an important role. When it comes to the AdS/CFT correspondence on the pp-wave background at the interacting level [23], the story becomes more complicated. It was questioned [24] whether supersymmetry alone can determine the whole string interaction vertex [25]. We would like to see whether our new symmetry can give further restrictions in its construction.

Another application is related to the Hubbard model. It was noticed [18, 26] that the AdS/CFT S-matrix is equivalent to Shastry's S-matrix. Understanding how this new hidden symmetry appears in the Hubbard model could help understanding more deeply the observed equivalence.

Finally, another interesting direction is to explore whether this symmetry could be embedded in the full $\mathfrak{psu}(2, 2|4)$ algebra before breaking it to $(\mathfrak{su}(2|2))^2$, when the ground state is identified. We reserve to return on these issues in future investigations.

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